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Determining the Elements that Govern the Occurrence of the Refracting Boundary Under the Assumppion that it is Planar only in the Region of Reception of Seismic Waves

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THE DETERMINATION OF THE ELEMENTS GOVERNING THE STRATIFICATION OF THE REFRACTING BOUNDARY ON THE HYPOTHESIS THAT IT IS FLAT ONLY IN A REGION WHERE SEISMIC WAVES ARE RECEIVED

Yu. V. Riznichko

Submitted by Academician O. Yu. Shmidt

The possibility of a quantitative interpretation of observations on refracted seismic waves is discussed, when the composition of the medium, on a path from the source of oscillation to the region of reception, is characterized by grave complexity. The relative composition of the medium on this path is not rendered by the hypothesis. It is only assumed that in the region of reception the refracting boundary is flat, but the boundary velocity and the velocity in the covering medium are constant. Under these conditions the angle and direction of incidence of the boundary in this region is determined. The general discussion on the variations of the problem and data on the complete solution of it is based on two observations.

1. Stating the Problem

The seismic waves (mainly longitudinal) which have been produced either by explosion or earthquake at point O_i are observed at point M_k (Figure 1a) where they arrive as refracted (head) waves. The series of individual observations at the various locations of points O_i and M_k ($i = 1, 2, \ldots; k = 1, 2, \ldots$) are used for the determination of elements of stratification -- the direction and the angle of incidence -- the refracting boundaries in a reception region (Figures 1a and 1b).

Each observation consists of determining the time of arrival t_i and gradient τ_i -- functions of a surface hodograph of a refracted wave, which proceeded from O_i to M_k. For this, it is practically necessary to make observations of the profile crosses intersecting at points M_k. Subsequently for this specific problem, we shall use only the dimensuration values of τ_{ik}, ignoring the values t_{ik}.

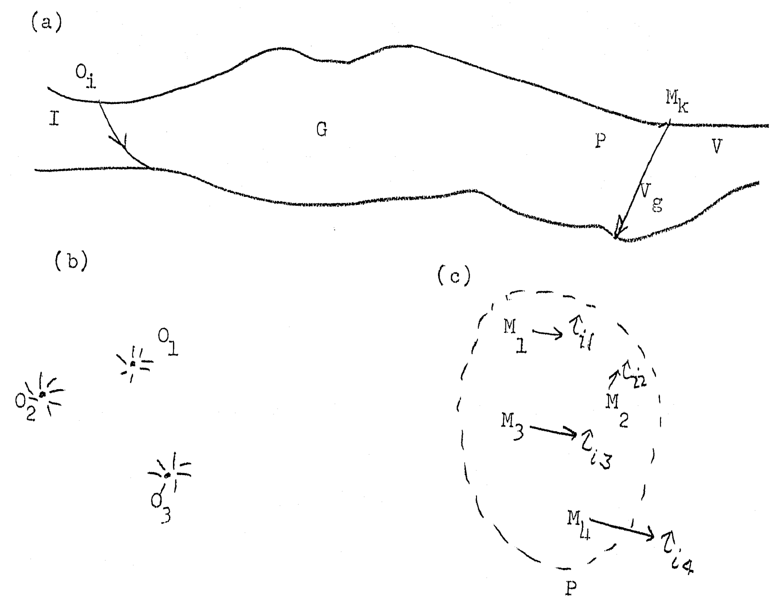

Figure 1. Stating the problem: (a) -- cross section; (b) plan; I is an area where the O_i points of the seismic wave emanation are located; P is an area where reception points M_k are located, -- here gradients τ_{ik} are determined as functions of a surface hodograph of refracted waves; G is an interjacent area; i. k = 1, 2, 3,...

Between area I (with points O_i) and area P (in which oscillation receivers are located) let there be an area G, where the conditions of passing waves are not known and are quite complex. The refracting boundary may differ considerably from a plane, the medium may be anisotropic, and the velocity of wave propagation in the medium above and below the boundary may be changed in any manner in space (down to the disappearance of the very boundary's sudden change in velocity). All these conditions, however, must be such that for a wave in a reception area P, in a layer under the boundary could be considered as a sliding wave.

At the same time, let the simple conditions exist in area P allowing the following ideal conditions: the section of the refraction boundary through which the refracted waves proceed to the points of reception M_k can be considered flat, the medium as isotropic, and the velocities V and V_g in the atmosphere, respectively above and below the refraction boundary, can be considered as constants (V_g is the "boundary" velocity at which the sliding wave runs lengthwise the boundary; $V_g > V$).

Then, it is natural to state the following problem: not setting up any assumptions regarding the structure of the medium in the area of sources I and the interjacent region G, and having assumed in respect to the area of reception P the conditions indicated above, to determine the elements governing the refractive boundary in its horizontal section in the reception area, using the series of observations t_{ik} in the profile crosses which are located in this area.

Let us note that in such a statement of the problem of the interpretation of observations according to the method of refracted waves, it differs from those elucidated in detail in literature (mainly in the works of I. S. Berzon [1, 2] and others) concerning refracted waves, when, in relation to the structure of the medium, definite hypotheses generally are set up not only for the reception area P, but also for other areas (I and G), lying on the path of oscillation propagation from the sources.

2. Idea and Qualitative Analysis of the Solution

Let us assume at first that the velocities V and V_g in the medium immediately above and below the refracting boundary are known constants. Then each observation of a vector τ_{ik} produced at point M_k permits a ready establishment of the corresponding direction r_{ik} of seismic rays arriving at this point (see, for example, [3], page 48).

Proceeding to a preliminary analysis of the problem, let us investigate the reciprocal disposition of vectors r_{ik} of rays and vectors n of the normal contemplated flat area of the refractive boundary in an area of reception P.

Let us fix an arbitrary point 0 as the origin. All vectors of the coordinate axes subject to analysis shall be related to this point, fixing at this point the origin of all vectors.

In the geometric locus of vectors r_{ik} there will be a surface of a circular cone with apex 0 and axis n upon which angle j (between the generatrix r_{ik} and axis n of this cone) is known:

$$\sin j = \frac{V}{V_g} \tag{1}$$

Having set up this relation, we proceed with the solution of the problem. Let us assume that the vectors r_{ik} are known and we shall look for the locus of vector n. If we should have at our disposal the infinite number of all possible vectors r_{ik}, then they would generate a complete surface of a circular cone as described above. This would permit determination of its axis n and produce a unique solution to the problem. But in view of a practical, limited number of observations, it is natural to proceed from the hypothesis that in our disposition there are only a limited number of vectors r_{ik}. Let us consider the possibility of a solution through different numbers of these vectors to which correspond a different number of individual observations.

1. If there is one vector r_{ik}, then the unknown vector n in conjunction with the previously given angle j, can revolve around 0, forming another conic surface with axis r_{ik}.

The problem has infinitely many solutions, depending on one parameter. If, in the capacity of an independent parameter, we select azimuth α as an incidence bearing of the boundary, then angle φ of boundary incidence would be determined as a function of this parameter: $\varphi = \varphi(\alpha)$.

2. If there are only two different vectors $r_{ik} = r_{\ell}$, $\ell = 1, 2$ (in Figure 2a vectors r_1 and r_2), then the conic surface with axis n may be superimposed on these vectors exactly as shown in Figure 2a. By this, the position of the axis n of the cone will be fixed, and the problem will be solved. More precisely, two positions of this cone are possible: by superimposition on the vectors r_1 and r_2 "on the left" and "on the right". In conformity with this two possible positions of the unknown vector n are determined (in Figure 2a only one of them is shown, namely, for the superimposition of a cone "on the left").

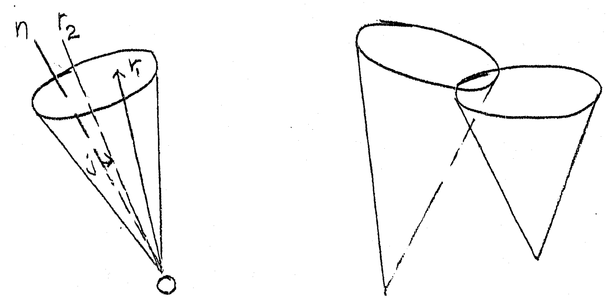

Figure 2. With the idea of solving the problem by two observations: (a) a cone whose axis is the normal n to the refracting boundary, and the generatrices of the cone are rays r_1 and r_2; (b) a cone whose axes are the rays r_1 and r_2; lines n_1 and n_2, intersections of the conic surfaces, determine the possible direction of the normal n to the refracting boundary.

The solution of this problem in the case of two vectors r_1 and r_2 may be obtained by means of the following three-dimensional construction; for each of these two vectors r_ℓ, is constructed a conic surface representing the geometric locus of all possible positions of vector n, corresponding to vector r_ℓ (Figure 2b). Two lines of intersection of these two conic surfaces give two of the positions possible (n_1 and n_2) of the unknown vector n. The structure may also be generated by means of the circumference of a sphere of unit radius.

Of the two possible variants of the determination of stratification elements of a refracting boundary which have been determined heretofore by the data in Paragraph 1 (Stating the Problem) only one variant makes physical sense. It is possible to select one, making use of additional physical and geological considerations. In such a way, the geophysical problem under investigation has a single (in the sense indicated) solution if there are two different vectors t_{ik}. These vectors can be obtained either from one explosion point O_i, but in two different points of observation M_k, or at one point M_k, but at two different points O_i, or, finally, by changing the position of both point O_i and point M_k.

It is easy to obtain a qualitative idea of the degree of stability of this problem in relation to the reciprocal distribution of the vector r_ℓ, assuming that each of the vectors may contain certain errors. For this we use the following graphic reasoning.

When vectors r_1 and r_2 draw together, tending to coincide one with another, then the degree of "reliability" of the superimposition of the cone with axis n (Figure 2a) is decreased, and a small variation in the position of any of them from vector r_ℓ will effect a great change in the position of the cone, and consequently, also in its axis n. This is indicative of a decreased stability in the solution of the problem.

At the point where $r_1 = r_2$ the problem is resolved to the preceding case where only one vector r_{ik} was established. By the waves observed at one origin O_i, which are produced in two points M_k (also in a greater number of points), for example, a flat front of a refracted wave in region P is indicated. In this case the problem does not have a unique solution.

By expanding vectors r_1 and r_2 from the position of reciprocal coincidence, the degree of superimposition "reliability" of the cone

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at first is increased, such that there is a corresponding increase in the reliability of the solution. However, on reaching a maximum for a sufficiently large span [pencil] of vectors r_1 and r_2, the reliability starts changing after this, and, when the angle between these vectors attains 2j, the reliability is lost: the cone can "slip" between vectors r_1 and r_2. This happens in the case when seismic waves come toward the observation area from opposite sides, more precisely, when the passing rays, corresponding to vectors r_1 and r_2, have directly opposite directions. In this case the practical problem also has no unique solution (the set of all possible solutions depends on one parameter).

Finally, if the angle between the vectors r_1 and r_2 becomes larger than 2j -- physically this is impossible -- then the problem has no solution at all.

Let us follow further the comparative divergence of the two vectors n_1 and n_2, which present two formally possible solutions to the problem of the fixed position of the normal n to the refracting boundary. From an examination of Figure 2a or 2b it follows that when vectors r_1 and r_2 approach superimposition, i.e. the angle between them approaches 0^o, then the vectors n_1 and n_2 are at maximum difference: the angle between them approaches 2j. When vectors r_1 and r_2 begin to separate, then vectors n_1 and n_2 come together.

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From the geophysical point of view, for obtaining the only practical solution to the problem, this drawing together (if it is excessive) is disadvantageous because there is a risk that both formally possible solutions may seem physically possible.

The drawing together of vectors n_1 and n_2 is completed as a total convergence when vectors r_1 and r_2 form an angle 2j. This being the case, however, there appear no possibilities of obtaining a unique solution to the problem because of another reason, shown above, namely, as a result of the reliability loss of the solution.

3. Let us assume now that there are three different vectors $r_{ik} = r_\ell$ ($\ell = 1, 2, 3$). Then from the formation of geometric figures similar to Figure 2a or Figure 2b, we obtain a unique solution to the problem: the unique position of the cone with axis n, folding over three vectors r_ℓ, or just a single line n intersecting all three conic surfaces with axis r_ℓ. The last case of the three vectors r_ℓ presents a plane, beginning with which the problem may be given essentially another character. By this it appears possible to state the problem in the following two ways.

(a) In the case of three (and of a greater number if $\ell \geqslant 3$) given vectors r_ℓ, it is possible to assume that each of them contained some error, and set up the problem of determining such conic surfaces (with angle n, r_ℓ = j) which by the best method would be approximated by the different positions of these vectors.

(b) In the case of three vectors r_ℓ it is possible by rearranging to compute the given parameter V_g and set up the problem of its computation simultaneously with n.

This problem is solved by a unique method. For constructions on a sphere of unit radius, it leads to drawing a circle through three given points, determined by the vectors r_ℓ; the radius of this circle determines V_g, and also the position of the center -- n. The addition of a number of fixed vectors r_ℓ over three ($\ell > 3$) per-

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4. In the case of four observed τ_{ik} first there appears the possibility of not even setting up a value for the parameter V, i.e. to assume that both velocities V and V_g are constants which depend both on observation and vector n. Increasing the number of given τ_{ik} to more than four permits us to solve this problem in order to obtain the most probable results.

However, the practical possibilities of a solution to the problem under consideration in its fullest scope is limited by the fact that it looses reliability in relation to the unknown value V, when the incident angle φ of the boundary tends to zero (see [4], page 90). In view of this, it is of no value to set up small φ angles. For still larger φ angles, when reliability exists, difficulties of a physical character appear, linked to the generally accompanying φ angles with complications of medium composition.: intense variability in the range of magnitude of φ, V and V_g. This either leads to the necessity of keeping track of the conditions of great "scattering effect" of observed values of τ_{ik} and proceeds to a decrease in the points of the result, or for forces giving up the assumption of constancy of the parameters φ, V, and V_g, which lie at the base of setting up every problem, and in general, eliminate the possibility of solution.

In such a way, in the case of four or more observations of τ_{ik} it is safer to set up the problem proceeding from the hypothesis that the magnitude of V is given, and to find either V_g and n, or only n, supposing that the magnitude of V_g is also given. The latter finally leads to greater reliability of the determination of n.

It may also be noted that in the desire to come as close as possible to the conditions that V, V_g, and φ are constants, it is necessary to attempt conscientiously to carry out the observations in a small area P, and for obtaining stable results -- to use as origins the O_i points which are situated in substantially different directions from this area. Then in area P it is possible to limit oneself to only one point O_i or, what is even better, when the position of O_i is changed to transpose point M_k also by such a calculation that the rays of refracted (head) waves passing through M_k will emerge approximately from one and the same element of the refracting boundary.

3. Calculations for the Case of Two Observations

Let us introduce for consideration a fixed system of coordinates -- such that the XY plane coincides with the plane of observation (or is parallel to it), and the Z axis is directed upwards. Let azimuths of the directed vectors be read from the XY plane to the X axis according to the direction from the Y axis.

If the mensurable vectors $\tau_{ik} = \tau_\ell$ (ℓ = 1, 2) have azimuths α_ℓ, then the component vectors r_ℓ are as shown in the formulas:

$$
\begin{aligned}
x_\ell &= \sin i_\ell \cos \\
y_\ell &= \sin i_\ell \sin \\
z_\ell &= \cos i_\ell
\end{aligned}
\qquad (2)
$$

where

$$\sin i_\ell = V |\tau_\ell|$$

(the meaning of all the terms was given in Paragraph 2).

Let us assume that vector n, as well as vectors r_ℓ, is unity, $n^2 = 1$. In conformance with Paragraph 2, item 2, the angles between vectors n and r_ℓ are known and are equal to j:

$$\sin j = \frac{V}{V_g}.$$

Let us set

$$\cos j = A.$$

Then we shall have the first equation for determination of vector n:

$$nr_1 = A, \quad (3)$$

$$nr_2 = A. \quad (4)$$

Each of these equalities corresponds to a family of cones with a fixed base r_ℓ and a fixed vertex angle. The determination of the lines of mutual intersection of the conic surfaces results in finding a vector n which satisfies both equations (Figure 2b), i.e. in the solution of the system of equations (3) and (4) with regard to n (when $n^2 = 1$).

Let us present (3), (4) and $n^2 = 1$ in coordinates:

$$x_1n_x + y_1n_y + z_1n_z = A, \quad (5)$$

$$x_2n_x + y_2n_y + z_2n_z = A, \quad (6)$$

$$n_x^2 + n_y^2 + n_z^2 = 1. \quad (7)$$

In this system of three equations the unknowns are the three coordinates n_x, n_y, n_z of the undetermined vector n. For their determination we find from (5) and (6) the unknowns n_x and n_y, expressing them as functions of n_z:

$$n_x = a_1 n_z + b_1, \tag{8}$$

$$n_y = a_2 n_z + b_2, \tag{9}$$

$$a_1 = \frac{y_1 z_2 - y_2 z_1}{x_1 y_2 - x_2 y_1} \qquad b_1 = A\frac{y_2 - y_1}{x_1 y_2 - x_2 y_1} \tag{10}$$

$$a_2 = \frac{x_2 z_1 - x_1 z_2}{x_1 y_2 - x_2 y_1} \qquad b_2 = A\frac{x_1 - x_2}{x_1 y_2 - x_2 y_1}$$

After this we substituted (8) and (9) in (7) as a result we obtain a quadratic equation in the unknown n_z.

$$(a_1^2 + a_2^2 + 1)n_z^2 + 2(a_1 b_1 + a_2 b_2)n_z + (b_1^2 + b_2^2 - 1) = 0 \tag{11}$$

Solving it, we finally arrive at

$$n_z = \frac{-(a_1 b_1 + a_2 b_2) \pm \sqrt{(a_1 b_1 + a_2 b_2)^2 - (a_1^2 + a_2^2 + 1)(b_1^2 + b_2^2 - 1)}}{a_1^2 + a_2^2 + 1} \tag{12}$$

The two roots of equation (11) correspond to two possible values (n_1 and n_2 of Figure 2) of vector n; the solution having physical significance is chosen on the basis of additional considerations (Paragraph 2, Section 2).

The remaining two coordinates of vector n we derive by substituting (12) in (8) and (9).

The elements governing the stratification of the refracting boundary are determined by these formulas (Figure 3):

$$\tan \varphi = \frac{\sqrt{n_x^2 + n_y^2}}{n_z} \qquad (13)$$

$$\tan \alpha = \frac{n_y}{n_x} \qquad (14)$$

Here φ is the angle of incidence and α is the azimuth of the direction of incidence of the refracting boundary in the area of oscillation reception which it was necessary to find.

Conclusion

The problem under investigation here can offer interest principally through a study of deep separation boundaries in the earth's crust by means of seismic waves from sufficiently distant explosions and from "near by" natural earthquakes. Moreover, the recording of refracted waves incited by artificial and natural sources and corresponding to one and the same boundary of separation, can be processed together disregarding the coordinates of the source of excitation.

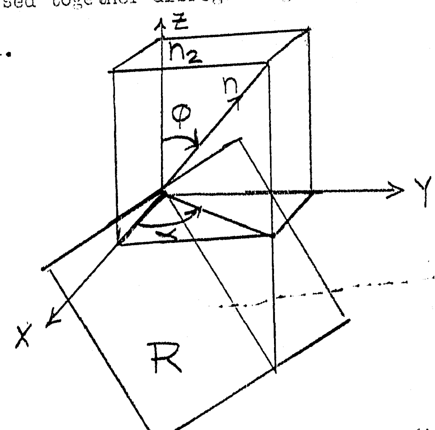

Figure 3. In the solution of the problem according to formulas (13),

(14): n is the normal to the refracting boundary R; φ is the angle of incidence, α is the azimuth of incidence of the boundary R.

The methods of interpretation which have been suggested here, apparently can find application to the methods of depth sounding of the earth's crust and to new correlation methods of earthquake observation which are being worked out in the Department of Experimental Seismology of the Geophysical Institute of AS USSR under the direction of G. A. Gamburtsev.

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Geophysical Institute

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